I P S B
INDIANA PROFESSIONAL STANDARDS BOARD

DRAFT Standards for Teachers of Mathematics

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101 West Ohio Street . Suite 300 . Indianapolis, IN 46204-1953
317/232-9010 . FAX 317/232-9023

STANDARDS FOR TEACHERS OF MATHEMATICS

Standard #1: Teachers of mathematics understand the key concepts and procedures of mathematics and have a broad understanding of the mathematics curriculum. Teachers of mathematics understand the appropriate structures within the discipline and its interaction with technology. (Refer to Addenda A and B.)

Performances

1. The teacher incorporates the historical perspective and changing nature of mathematics in his/her approach to teaching students.

2. The teacher represents and uses differing viewpoints, theories, "ways of knowing," and methods of inquiry in his/her teaching of mathematical concepts.

3. The teacher evaluates teaching resources and curriculum materials for comprehensiveness, accuracy, and usefulness for representing particular concepts of mathematics.

4. The teacher uses mathematical concepts, assumptions, tools of inquiry (problem solving), and reasoning's that are central to the discipline.

5. The teacher engages students in generating knowledge to formulate and test hypotheses according to the methods of inquiry and standards of evidence used in mathematics.

6. The teacher engages students in meaningful experiences that extend their understanding of mathematics as a network of interconnected concepts.

7. The teacher integrates the use of technology into the mathematics classroom.

8. The teacher integrates knowledge, skills, and methods of inquiry from other subject areas into the teaching of mathematics.

Knowledge

1. The teacher knows the history of mathematics, the dynamic nature of mathematics, and the changing ways we learn, teach, and do mathematics.

2. The teacher understands the role of technology in mathematics.

3. The teacher understands major concepts, assumptions, tools of inquiry (problem solving), and mathematical reasoning that are central to the discipline.

4. The teacher understands how students' conceptual frameworks and their conceptions/mi

5. The teacher knows and understands relevant mathematical connections within the discipline, the real-world context, and other subject areas.

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Dispositions

1. The teacher appreciates that mathematics is dynamic and ever-evolving from looking at the historical and current development of mathematical thought, including perspectives involving ethnic diversity.

2. The teacher values the integration of technology with mathematics.

3. The teacher values the critical linkages among problem solving, communication, connections, and technology in mathematics.

4. The teacher appreciates multiple perspectives of how learners develop mathematical knowledge from their own vantage.

5. The teacher possesses enthusiasm for mathematics and values connecting mathematics to other fields and to everyday life.

6. The teacher values a conceptual framework of mathematical understanding which leads to students' construction of mathematical knowledge.

Standard #2: Teachers of mathematics understand how students learn mathematics and provide learning opportunities that support their intellectual, social, and personal development.

Performances

1. The teacher assesses individual and group performance, growth, and achievement and designs strategies that meet learners' needs in each domain and lead to the next level of development.

2. The teacher stimulates student reflection on prior knowledge and uses instructional strategies for active engagement, manipulation, and testing of ideas in order to extend and enrich the mathematical learning experience.

3. The teacher uses current theories and research from mathematics education to make instructional decisions related to students' developmental levels.

4. The teacher effectively uses multiple representations and explanations of mathematical concepts that encourage students to see, question, interpret, and discover ideas from diverse perspectives

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Knowledge

1. The teacher understands how the learning of mathematics occurs, how students construct mathematical knowledge, and how to use instructional strategies that promote student learning.

2. The teacher understands how students' physical, social, emotional, and cognitive development influences the learning of mathematics.

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3. The teacher maintains an awareness of expected developmental progressions and ranges of individual variation within each domain (physical, social, emotional, and cognitive), identifies levels of readiness in the learning of mathematics, and understands how development in any one domain may affect performance in others.

Dispositions

1. The teacher appreciates how the learning of mathematics occurs through the use of instructional strategies that promote student learning.

2. The teacher values individual variation within each area of development, respects the diverse talents of all learners, and commits to students' development and confidence.

3. The teacher appreciates students' strengths as a basis for constructing mathematical knowledge and their errors as an opportunity for clarification of students' misconceptions and discovery of mathematical concepts.

Standard #3: Teachers of mathematics understand how students differ in their approaches to learning and create instructional opportunities that are adapted to diverse learners.

Performances

1. The teacher identifies, designs, and provides for mathematics instruction appropriate to students' stages of development, learning styles, strengths, and needs.

2. The teacher accesses appropriate services or resources to meet exceptional mathematical learning needs of students.

3. The teacher seeks and uses information about students' backgrounds, cultures, language acquisition, and communities to bring multiple perspectives to the study of mathematics and to link mathematics instruction.

4. The teacher creates a learning community in which individual differences are respected.

5. The teacher creates opportunities for all students to learn challenging mathematics successfully.

Knowledge

1. The teacher understands differences in approaches to learning and performing mathematics, including different learning styles, multiple intelligences, and performance modes.

2. The teacher knows about areas of exceptionality in learning--including learning disabilities, visual and perceptual difficulties, gifted and talented, and other physical/mental challenges.

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3. The teacher understands the process of second language acquisition and strategies which support the learning of students whose first language is not English.

4. The teacher understands how students' learning of mathematics is influenced by individual experiences, talents, prior learning, language acquisition, culture, family, and community values.

5. The teacher understands cultural and community diversity and knows how to incorporate students' experiences, culture, and community resources into mathematics instruction.

6. The teacher understands that all children can learn mathematics successfully.

Dispositions

1. The teacher commits to posing mathematics tasks, inspiring inquiry, and encouraging individual excellence.

2. The teacher respects students as individuals with differing personal and family backgrounds and various skills, talents, and interests.

3. The teacher respects the challenges that students face in second language acquisition and the need for strategies which support the learner whose first language is not English.

4. The teacher values a sensitivity to community and cultural norms.

5. The teacher believes all children can learn mathematics at a challenging level.

Standard #4: Teachers of mathematics understand and use a variety of instructional strategies to encourage students' development of critical thinking, problem-solving, and performance skills. (Refer to Addendum A.)

Performances

1. The teacher uses problem solving and reasoning processes as the basis for mathematical inquiry and incorporates a variety of instructional strategies to engage students in critical thinking and discovery activities.

2. The teacher carefully evaluates how to achieve curriculum standards of mathematical learning, choosing alternative forms of instruction, assessment strategies, materials, and technological tools that meet the conjectures, interests, developmental stages, prior knowledge, and learning styles of students.

3. The teacher enhances learning through the uses of various resources such as computers, calculators, concrete materials, manipulatives, models, and other technological representations.

4. The teacher monitors and adjusts instructional and assessment strategies in response to students' leads, conjectures, and discoveries.

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Knowledge

1. The teacher understands problem solving and the reasoning process as the basis for mathematical inquiry and knows a variety of instructional strategies (such as questioning techniques, tasks that elicit and challenge student discovery, and problem formulation) to encourage critical thinking.

2. The teacher understands alternative strategies such as cooperative and team learning, whole group discussion, and constructive learning as a foundation to create a mathematical community of teacher-student or student-student discourse that engages students in reflective processes.

3. The teacher knows how to enhance learning through the use of a variety of resources such as computers, calculators, concrete materials, manipulatives, models, and other technological representations. (Refer to Addendum B.)

Dispositions

1. The teacher values the problem-solving and reasoning process as the basis for mathematical inquiry and discovery.

2. The teacher values flexibility of instructional strategies and reciprocity in the mathematical community as necessary for adapting instruction to student needs for optimal learning.

3. The teacher values multiple teaching and learning strategies that actively engage students in constructive
learning opportunities that promote the development of critical thinking.

4. The teacher values the integration of technology into instruction.

Standard #5: Teachers of mathematics use an understanding of individual and group motivation and behavior to create a learning environment that encourages positive social interaction, active engagement in learning, and self-motivation.

Performances

1. The teacher engages students in individual and cooperative mathematics activities, analyzes the mathematics classroom environment, and makes adjustments to enhance student interaction, motivation, and investigative work.

2. The teacher helps the group to develop shared expectations for student interaction and creates a positive classroom climate of openness, mutual respect, support, and inquiry.

3. The teacher organizes, allocates, and manages the resources of time, space, and activities in order to make provisions for active and equitable engagement of all students in

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mathematical tasks.

4. The teacher places responsibility for learning on the student.

Knowledge

1. The teacher knows about human motivation, behavior, the nature of mathematics, and the way students learn mathematics individually and in groups.

2. The teacher understands how diverse groups interact and influence individuals.

3. The teacher understands the principles of effective classroom management which include the promotion of individual responsibility, positive relationships, and cooperation, all for purposeful learning in the mathematics classroom.

Dispositions

1. The teacher recognizes the importance of intrinsic motivation to students' lifelong mathematical growth.

2. The teacher values the role of students in others' learning, recognizes the importance of positive peer relationships, and encourages a mathematical community with interaction, discourse, and independent and collaborative student work.

3. The teacher values a well-managed classroom in which a positive climate of trust contributes to the continual development of individual student's mathematical skills.

4. The teacher believes that it is important for students to take responsibility for their own learning.

Standard #6: Teachers of mathematics use knowledge of effective verbal, nonverbal, and media communication techniques to foster active inquiry, collaboration, and supportive interaction in the classroom.

Performances

1. The teacher models effective verbal and nonverbal communication strategies in conveying mathematical ideas, information, and inquiry.

2. The teacher actively listens, solicits, supports, and expands learner expression in speaking, writing, and presenting mathematical models and representations.

3. The teacher communicates mathematics in ways that demonstrate and foster a sensitivity to cultural and gender differences among all students in the classroom.

4. The teacher uses technology and a variety of manipulatives, tools, models, and mathematical representations to communicate mathematically.

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Knowledge

1. The teacher understands communication techniques (verbal and nonverbal), the development of the language of mathematics, and the role of language in learning mathematical concepts.

2. The teacher knows how cultural and gender differences can affect the communication of mathematics in the classroom.

3. The teacher knows how to communicate mathematical concepts through the use of manipulatives, tools, models, symbols, graphic displays, and technology. (Refer to Addendum B.)

Dispositions

1. The teacher recognizes the power of language for fostering self-expression, active inquiry, justifications for mathematical resolutions, and the learning of mathematical ideas.

2. The teacher appreciates the role of the active listener as a facilitator.

3. The teacher appreciates the influences of students' linguistic, ethnic, racial, and gender diversities on learning.

4. The teacher values many ways in which people seek to communicate the many modes and representations for the process of mathematics.

Standard #7: Teachers of mathematics plan instruction based upon knowledge of subject matter, students, the community, and curriculum goals.

Performances

1. The teacher, as an individual and colleague, plans instruction based upon long- and short-term goals with regard to students' prior knowledge, preconceptions, and inquiry that is encouraged through exploration, problem solving, reasoning, and communication.

2. The teacher selects and creates real world mathematical tasks that are appropriate for curriculum goals, standards, and proficiencies as well as for the varied learning styles and developmental levels of students.

3. The teacher, individually and collegially, selects and creates tasks that integrate mathematics with other subject areas.

4. The teacher creates, evaluates, and adjusts plans in relation to student leads, performance, and needs

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in order to maximize student confidence, flexibility, discovery, perseverance, curiosity, and inventiveness in doing mathematics.

Knowledge

1. The teacher understands mathematics as a discipline of interconnected concepts, understands mathematical connections to other subject areas, and understands students' mathematical thinking and development, all as a basis for instructional planning.

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2. The teacher understands curriculum development in relation to instruction that aligns with curriculum standards, goals, and essential skills as outlined in the Indiana Department of Education Mathematics Proficiency Guide, National Council of Teachers of Mathematics Curriculum and Evaluation Standards of School Mathematics, Professional Standards for Teaching Mathematics, and Assessment Standards for School Mathematics.

3. The teacher knows the contextual considerations of instructional materials, individual student interests, needs, and aptitudes, and community resources that influence effective instructional planning connected to students' mathematical experiences and daily living.

4. The teacher knows when and how to adjust plans based on student responses, leads, conjectures, discoveries, prior knowledge, and inquiry.

5. The teacher knows how to plan cooperatively in order to develop effective mathematics curriculum and instructions.

Dispositions

1. The teacher values both long- and short-term planning as a consideration in regarding the continuum of mathematics curriculum across all developmental levels.

2. The teacher believes, as a researcher in the classroom, that plans must reflect students' developmental needs, the relevance and dynamic nature of current mathematics to the real world, and the current national and state standards.

3. The teacher values planning for the integration of mathematics with other subject areas.

4. The teacher values planning as a collegial activity.

Standard #8: Teachers of mathematics understand and use formal and informal assessment strategies to evaluate and ensure the ongoing intellectual, social, and personal development of the learner.

Performances

1. The teacher selects, constructs, and uses a variety of developmentally appropriate formal and informal assessment techniques to determine students' understanding of mathematics and to modify teaching strategies.

2. The teacher evaluates the effect of mathematics tasks on individuals and groups collectively.

3. The teacher uses an ongoing process of assessment to adjust instructional plans for optimal learning, to assure the assessment of what is being taught, and to align instructional practice to the curriculum goals, standards, and proficiencies.

4. The teacher communicates student progress knowledgeably and responsibly, based on

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appropriate indicators, to students, parents, families, and colleagues.

5. The teacher uses assessment strategies that involve the learners in self-assessment activities to help them find their strengths and needs.

Knowledge

1. The teacher understands the characteristics, uses, advantages, and limitations of different types of assessments (i.e., Appendix A) for evaluating what students know and are able to do.

2. The teacher understands a variety of assessment techniques to determine what mathematical tasks will support student growth and development.

3. The teacher understands how to select, construct, communicate, and use assessment strategies in alignment with curriculum standards, goals, and instruction so that what is being taught is being assessed.

4. The teacher understands assessment-related issues such as validity, reliability, bias, rubrics, portfolios, alternative assessments (i.e., Appendix A), and other formative and summative assessments.

5. The teacher knows that ongoing assessment is essential to an instructional process which adapts to student needs, learning styles, and developmental readiness.

6. The teacher knows that assessment techniques may be used to promote and develop students' ability to reflect upon their own learning and become self-reliant learners.

Dispositions

1. The teacher values ongoing assessment and recognizes that many assessment strategies, when accurately and systematically used, are necessary for monitoring and promoting student learning.

2. The teacher appreciates the use of a variety of assessments to identify and communicate student strengths in the knowledge of mathematics, to direct students to access optimal learning opportunities, to align instruction with curriculum goals, standards, and proficiencies, and to address students' developmental readiness.

3. The teacher values the task of the learner engaging in self-assessment.

Standard #9: Teachers of mathematics are reflective practitioners who continually evaluate the effects of their choices and actions on others (students, parents, families, and other professionals in the learning community) and who actively seek out opportunities to grow professionally.

Performances

1. The teacher uses classroom observation, student performance, and research as sources for

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experimenting with, reflecting on, and revising instructional practices specific to mathematics.

2. The teacher examines and revises his/her assumptions about the nature of mathematics and how it is learned and should be taught, and then experiments thoughtfully with alternative strategies in the classroom.

3. The teacher revisits school mathematics for deeper understandings of the concepts and seeks other resources such as community/business/university partnerships to support his/her own development as a lifelong learner of mathematics.

4. The teacher draws upon professional literature, colleagues, and learned societies as supports for reflection, problem solving, new ideas, sharing experiences, and participating in workshops and courses related to mathematics.

5. The teacher advocates and models improved practices for the teaching of mathematics among colleagues through professional development opportunities, post-graduate course work, and the sharing of professional resources.

Knowledge

1. The teacher understands a variety of self-assessment and problem-solving strategies related to student success and his/her instructional practices.

2. The teacher knows current research and resources for professional learning (i.e., Appendix B) in the teaching of mathematics.

3. The teacher knows his/her professional responsibility to be an ongoing, self-directed learner of mathematics so as to continually develop and refine practices that attend to the mathematical needs of students.

4. The teacher knows his/her professional responsibility to advocate improved practices for the teaching of mathematics among colleagues.

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Dispositions

1. The teacher values reflection, self-assessment, and self-directed learning as an ongoing process.

2. The teacher values the search, development, and continual refinement of practices that address the individual needs of the mathematics learner.

3. The teacher views him/herself as a mathematics educator who engages in and supports appropriate practices for self and colleagues as guided by professional affiliations.

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Standard #10: Teachers of mathematics foster relationships with school colleagues, parents, families, and agencies in the larger community to support student learning and well being.

Performances

1. The teacher participates in interdisciplinary/collegial activities designed to make the entire school a productive learning environment for mathematics education.

2. The teacher connects to the students' other environments by consulting with parents, families, counselors, other teachers, staff members within the schools, and business/community partners.

3. The teacher identifies and uses community resources to foster student learning and connect mathematics with daily living.

4. The teacher actively listens to the student and responds to cues of distress, investigates situations, and seeks appropriate outside help to remedy problems.

5. The teacher participates in school and community efforts to effect positive change in mathematics education and the learning environment.

Knowledge

1. The teacher understands schools as organizations within the larger community and understands the operations of the relevant aspects of the system(s) within which s/he works.

2. The teacher understands how factors in the students' environment outside of school may influence students' life and learning.

3. The teacher understands and abides by laws and policies related to students' rights and teacher responsibilities.

4. The teacher understands how to develop collegial relationships and create interdisciplinary learning within the school community.

Dispositions

1. The teacher values the importance of all aspects of a child's life, experience, and well being
(cognitive, emotional, social, and physical).

2. The teacher values consultation with parents, families, other educators, business/community partners, and members of the larger community as a resource for the education and well-being of the student.

3. The teacher respects the privacy and confidentiality of student concerns and information.

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4. The teacher appreciates his/her role as an advocate for students.

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Addendum A: Developmental Level Content Requirements

In recognition of instructional practices reflective of developmental learning, the following levels are addressed:

Early Childhood
Middle Childhood
Early Adolescence
Adolescence and Young Adult

The teacher of mathematics develops a knowledge of mathematics through the following critical linkages:

Problem Solving in Mathematics
Communication in Mathematics
Reasoning in Mathematic
Mathematical Connections
Technology in Mathematics

The teacher of mathematics understands the following mathematical content:

Mathematics for All Teachers

Foundational knowledge in mathematics is essential for those teaching mathematics at any level.

With regard to specific content preparation, the mathematical education of all teachers should include the following:

Number systems and number sense. Teachers of mathematics should have a well-developed numbers sense (including mathematics, estimation, and reasonableness of results) and an understanding of the use of number concepts, operations, and properties (including basic number theory), of the role of algorithms, and of place value. In setting the view of these ideas in the curriculum, teachers should be able to extend the number systems from the whole numbers to fractions and integers, and then to rational and real numbers, while including a discussion of the extension of the operations, properties, and ordering. Notions of fractions, decimals, percents, ratio, and proportion should be developed through problems with an applied flavor.

Geometry. Teachers of mathematics should understand how geometry is used to describe the world in which we live and how geometry can be used to solve real-world problems. Analysis of two- and three-dimensional figures should include the study of tessellations, symmetry, polygons, polyhedra, and curved shapes. Synthetic, coordinate, and transformational geometry should be used to provide opportunities for teachers to solve problems and to hone their skills in building justifications and coherent arguments for the plausibility of conjectures. Throughout the experience, spatial visualization should be emphasized.

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Measurement. The concept of measurement needs to be understood from the perspective of its historical development. The attributes of what we measure include length, area, volume, capacity, time, temperature, angles, weight, and mass. Teachers should understand that the units to record measure are different from the process of measurement itself. These ideas should be reinforced through varied experiences, using both standard and nonstandard units where teachers learn to estimate lengths, areas, and so on. Of particular importance should be an understanding of the System International d'Units (the metric system). Derivations of the formulas for the perimeter, area, and volume of common figures should be approached through meaningful explorations. Indirect measurement and its many applications should be studied.

Statistics and probability. Teachers should have a variety of experiences in the collection, organization, representation, analysis, and interpretation of data. Key statistical concepts for all teachers include measures of central tendency, measures of variation (range, standard deviation, interquartile range, and outliers), and general distributions.

Representations of data should include various types of graphs, including bar, line, circle, and pictographs, as well as line plots, stem-and-leaf plots, box plots, histograms, and scatter plots. Probability of simple and compound events and its use in quantifying uncertainty should be built into these experiences.

Teachers should have opportunities to explore empirical probability from simulations and from data they have collected and to analyze theoretical probability on the basis of a description of the underlying sample space. Probability trees and simulations using objects such as spinners, dice, slips of paper, and so on should be used to solve problems.

Functions and use of variables. Teachers need to experience the development of mathematical language and symbolism and how these have influenced the way we communicate mathematical ideas. Also, experience in representing and solving problems requiring the use of variables is important. To build bridges for their students to the mathematics that comes late in the school curriculum, teachers must have a basic understanding of the concepts of functions and their use in the growth of mathematical ideas. Understanding different representations of functions (tabular, graphical, symbolic, verbal), how to move among these representations, and the strengths and limitations of each is fundamental. The distinction between continuous and discrete approaches in the solution of mathematical problems should also be a part of the experiences provided by these teachers and should be introduced initially at an intuitive and informed level.

Problem Solving. Using critical thinking teachers use what they have learned (concepts and existing data) to find out what they do not know and instruct students in this process. Teachers use mathematical inquiry, including questioning techniques, discovery, reasoning process, identifying strategies, reflective process, analysis and justification, formulating the problem and moving from simple ideas to complex.

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Additional Mathematics for Teachers of Early Adolescents, Adolescents and Young Adults

Teachers of mathematics at these levels must present mathematics that builds on the students' background. New mathematical knowledge should deepen the understandings of the topics already noted and introduce new and worthwhile mathematical ideas. With regard to specific content preparation, the mathematical education of teachers at these levels should include and extend the material described earlier by including the following:

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Number systems and algebraic structures. The system of real numbers should be extended to the complex numbers. Investigations of selected algebraic structures should include concrete examples such as clock arithmetic, modular systems, and matrices. The properties of the operation in these structures and how they are reflected in the number systems of school mathematics should be investigated, especially the use of matrices and matrix operations to record information and to deal with solutions of systems of equations.

Geometry and measurement. Geometry should focus on intuitive, "common sense" investigations of geometric concepts in such a way that general properties emerge and are used as the basis for conjectures and deductions. Later, observations and deductions can be studied more formally as part of a mathematical system.

Tessellations, symmetry, congruence, similarity, measurement, trigonometry, and other notions can be investigated through two- and three-dimensional physical models, drawings, and computer graphics, emphasizing visualization.

Synthetic, coordinate, and transformational geometry should be revisited with an emphasis on solving problems. The need for assumptions, for more formal arguments, and for formulating, testing, and reformulating conjectures becomes more evident. Taxicab geometry and geometry on the sphere can be used to study alternatives to Euclidean plane geometry. Dimensional analysis can be used to solve more complex problems involving measurement and attendant conversions.

Statistics and probability. Teachers should learn to use key concepts of descriptive statistics, culminating in personal research projects that include experiences in collecting, organizing, analyzing, and interpreting data and in communicating the results of descriptive statistics to others. The concepts of dispersion and central tendency should be represented using techniques from exploratory data analysis. Relationships between two variables should be represented with scatter plots, and visual techniques for approximating a line of best fit through a scatter plot should be introduced as well. Potential misuses of statistics and common misconceptions of probability should be discussed. The power of simulation as a problem-solving technique for making decisions under uncertainty should be a prominent experience. Experiments involving dice, spinners, random numbers, and computer programs should be used to simulate probability and statistics problem situations. Other topics that should be introduced include fair games and expected value, odds, elementary counting techniques, conditional probability, and the use of an area model to represent probability geometrically.

Concepts of calculus. Teachers should acquire conceptual knowledge of the process of differentiation and integration, including examples of applications of these ideas in the sciences and in modeling and solving problems in mathematics. Functions, graphs, and the notion of limits should be explored, starting with concrete problems such as maximizing the volume of a box than can be folded from a rectangular sheet of grid paper. The concepts of limit and infinity should also be explored for their role in the history of the development of calculus and in the study of geometry.

Additional Mathematics for Teachers of Adolescents and Young Adults

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Teachers of mathematics at these levels build on the prior knowledge of students, provide students with broad experiences in the range of applications of mathematics, and help students extend and formalize their thinking and reasoning. With regard to specific content preparation, the mathematical education of teachers at these levels should include and extend the material described earlier by including the following:

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Number systems, number theory, algebra, and linear algebra. Further study of the system of complex numbers should include both geometric (vector) and polar representations of complex numbers and the interpretation of complex solutions to equations. Also, investigations of selected algebraic structures should include groups, rings, integral domains, and fields (including order relations). Topics in number theory should be explored, including modern topics such as coding theory. Because of its wide application, linear algebra should receive extensive treatment. In addition, functions acting on these structures, such as isomorphisms of groups and linear (matrix) functions acting on vector spaces should be investigated.

Geometry. Geometry should be extended to include vector geometry and additional work in synthetic, coordinate, and transformational geometries. Alternatives to the parallel postulate provide opportunities to reveal non-Euclidean geometries. An introduction to the foundations of geometry can provide teachers with insight into the power of the axiomatic method. The study of geometric transformations, an important manifestation of the function concept, shows the interplay between algebra and geometry. Experiences with linear algebra should be applied to the study of matrix representations of transformations and can shed light on the geometric effects of transformations and the algebraic structure of a set of transformations.

Statistics and probability. The study of probability and statistics should include both descriptive and inferential statistics and probability from both experimental and theoretical viewpoints. The theoretical probability and statistics should include both discrete and continuous probability distributions and should use such distributions to make inferences about populations. On the experimental side, teachers should have extensive experiences using and creating simulations of probability and statistical experiments, both with concrete objects such as dice and spinners and with computer programs. Misuses of statistics and common misconceptions of probability should be discussed. Descriptive statistics should include exploratory data analysis, including the median fit line for a scatter plot, as well as the traditional measures of dispersion and central tendency. Other statistics topics should include confidence intervals, hypothesis testing, correlations, and regression.

Calculus and analysis. Teachers should have a firm conceptual grasp of the notions of limit, continuity, differentiation, and integration and a thorough background in the techniques and applications of calculus. The development and use of calculus to model and solve problems involving rates of change, optimization, and measurement need to be appreciated as fundamentally important intellectual achievements in the history of mathematics.

Discrete mathematics. The tools and modeling processes of discrete mathematics have gained increased prominence in applications to real-world problems, including those in computer science. Thus, it is essential that the mathematical background of secondary school mathematics teachers include attention to symbolic logic, induction and recursion, relations, equivalence relations and functions, and sequences and series. A wide range of modeling applications of graphs and trees should be explored, along with properties of graphs and trees, matrix representations of graphs, and incidence paths in graphs. Other topics should include difference equations and an introduction to combinatorics.

Conclusion

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Given the nature of mathematics and the changes being recommended in the teaching of mathematics, teachers at all levels need substantive and comprehensive knowledge of the content and discourse of mathematics. In addition, teachers need to view mathematics through a variety of lenses, including the role and impact of culture, society, and technology and the place of school mathematics within the discipline of mathematics.

Addendum B: The Role of Technology and Concrete Models in Mathematics[1]

Understanding the history and the structure of mathematics is knowing that mathematics is not a static discipline. Even as you are reading this report, the uses of mathematics are changing. The search for solutions to new problems continues to create new mathematics. Teachers know the history of the development of mathematics and remain cognizant of the ever changing nature of the discipline. Many of the current changes in mathematics result from changes in technology. Teachers understand the changing ways in which we learn, teach, and do mathematics. These changes are a part of their understanding of and approach to mathematics. The National Council of Teachers of Mathematics Professional standards state:

> "*Technology changes the nature and emphasis of the content of mathematics as well as the pedagogical strategies used to teach mathematics. Performing computational and representational procedures by hand is time-consuming, and students often lose sight of mathematical insights or discoveries as they become mired in the mechanics of producing the results.*
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> *With the introduction of technology, it is possible to de-emphasize algorithmic skills; the resulting void may be filled by an increased emphasis on the development of mathematical concepts. Technology--computer and calculators--saves time and, more important, gives students access to powerful new ways to explore concepts at a depth that has not been possible in the past." (NCTM, p. 134)*

Teachers of mathematics integrate technology with mathematics in their approach to mathematics instruction. Technology makes some old mathematics unnecessary. Complicated calculations, repetitive graphing, long division, can now be completed using calculators, allowing students and teachers to explore the mathematical ideas that underlie the mechanics of computation. The emphasis should be on using computers and calculators as tools to represent mathematical ideas and construct different representations of mathematical concepts.

Computers are used to organize, represent, and analyze data, to conduct simulations, and to make predictions. Through the use of technology students can collect and organize data, such as recycling at the school, can graph and interpret data, and can analyze patterns. Technology keeps the focus on the mathematical concepts, not the laborious crunching of numbers. The use of technology also opens mathematical investigation to students at younger ages.

Teachers of mathematics also integrate a variety of non-technological tools into their instruction to help students explore and learn mathematics. Drawings, diagrams, analogies, and invented

[1]INTASC Model Standards in Mathematics for Beginning Teacher Licensing & Development

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symbols provide a vehicle for connecting mathematical ideas. Teachers are able to use concrete materials, such as tangrams, base-ten blocks, fraction bars, attribute blocks, counter, etc., as models to help students develop mathematical concepts and solve problems. Through these tools, abstract concepts can be explored in a concrete way, allowing students to explore mathematics at an earlier age.

Mathematics instruction should encourage students to select appropriate tools from technology, concrete materials, drawings, and diagrams based on what they find most useful for a given problem. Teachers of mathematics know when to introduce new tools that are appropriate for particular concepts or problems and that will expand a student's repertoire of problem-solving tools.

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